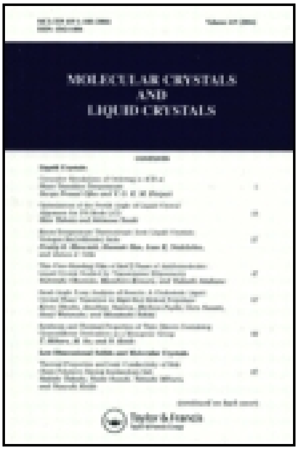

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Mykhailo Lednei [a], Igor Pinkevich [a], Victor Reshetnyak [a] & Timothy Sluckin [b]

[a] Kyiv Taras Shevchenko University, Physics Faculty, Prosp.Glushkova, 6, Kyiv, 252022, Ukraine

[b] University of Southampton, Faculty of Mathematical Studies, Southampton, S017 1BJ, United Kingdom

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Rayleigh-Gans Theory of Light Scattering in Filled Nematics

MYKHAILO LEDNEI[a], IGOR PINKEVICH[a], VICTOR RESHETNYAK[a] and TIMOTHY SLUCKIN[b]

[a]*Kyiv Taras Shevchenko University, Physics Faculty, Prosp.Glushkova, 6, Kyiv, 252022, Ukraine and* [b]*University of Southampton, Faculty of Mathematical Studies, Southampton S017 1BJ, United Kingdom*

The differential cross-sections of the light scattering by the director inhomogeneity near small spherical and cylindrical particles in nematic liquid crystals are calculated in the Rayleigh-Gans approximation. We use the Percus-Yevick approximation to account for interference effects. The influence of the boundary conditions at the particle surface as well as of external electric field on the angular dependence and magnitude of the light scattering cross-section is studied.

Keywords: filled liquid crystals; light scattering

1. INTRODUCTION

There has been much recent interest in filled liquid crystals (LC), in particular in filled nematics [1]. These materials consist of silica colloidal particles of diameter (approximately) 100A in a nematic liquid crystal suspension. The particles induce strong defects in the local nematic texture, which enhance their light scattering properties. In an external electric field the interaction between the texture and electric field can induce strong changes in optical properties that may result in display applications.

In this paper we present results of theoretical studies of light scattering by colloidal inclusions in filled nematics. We suppose that multiple scattering can be ignored and that the only many-scatterer effects are due to silica particle correlations. We also suppose that the light scattering by a silica particle is dominated by the director inhomogeneity it causes, and the direct scattering process can be ignored. Finally we use the Rayleigh-Gans approximation.

II. SCATTERING GEOMETRY AND BASIC FORMULAS

Let undisturbed nematic director and wave vector $\vec{k}$ of the incident light be directed along the axis *z* of Cartesian frame. The *yz*-plane is the plane of the incident and scattered ($\vec{k}'$) wave vectors (See Figure 1).

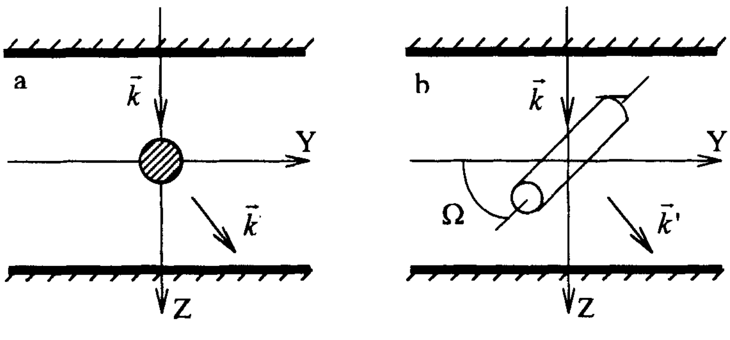


FIGURE 1. a - spherical particles, b - cylindrical particles.

In Rayleigh-Gans approximation the light scattering differential cross-section is given by the expression [2]

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{16\pi^2}\left|\vec{i}\,\hat{\varepsilon}(\vec{q}_s)\vec{f}\right|^2, \tag{1}$$

where

$$\hat{\varepsilon}(\vec{q}_s) = \iiint\left(\hat{\varepsilon}(\vec{r}) - \hat{\varepsilon}^0\right)\exp(-i\vec{q}_s\vec{r})dV, \qquad \vec{q}_s = \vec{k}' - \vec{k}. \tag{2}$$

Here $\vec{i}, \vec{f}$ are the unit vectors denoting the polarization of the incident and scattered light waves, $\hat{\varepsilon}(\vec{r})$, $\hat{\varepsilon}^0$ are the dielectric susceptibility tensors of the disturbed and undisturbed nematic, respectively. The integration is carried out over the volume of nematic neglecting the small volume occupied by the particles.

Assume the particles concentration to be small enough so the regions of director disturbance by different particles do not overlap. Then one can write

$$\varepsilon_{ij}(\vec{r}) = \varepsilon_{ij}^0 + \sum_m \delta\varepsilon_{ij}(\vec{r} - \vec{r}_m, \Omega_m)\Theta\left(d - |\vec{r} - \vec{r}_m|\right), \tag{3}$$

where d is an average distance between centers of particles, the function $\Theta(x) = 1$, if $x \geq 0$ and $\Theta(x) = 0$, if $x < 0$, Ω_m denotes the orientation of the particle (if cylindrical) in the *xy*-plane; the summation is over all N particles. One can calculate $\delta\varepsilon_{ij}(\vec{r} - \vec{r}_m, \Omega_m)$, using the next expression for dielectric susceptibility tensor of nematics [3]

$$\varepsilon_{ij}(\vec{r}) = \varepsilon_{\perp}\delta_{ij} + \varepsilon_a n_i(\vec{r}) n_j(\vec{r}), \tag{4}$$

where $\varepsilon_a = \varepsilon_{\parallel} - \varepsilon_{\perp}$ is the anisotropy of nematic dielectric susceptibility at the optical frequencies, $\vec{n}$ is a nematic director calculated from the condition of minimization of the filled nematic free energy.

On substituting (3) into the formula (1) and performing some obvious calculations we get in the case of cylindrical particles in nematics the next expression

$$\frac{d\sigma}{d\Omega}=\frac{Nk^4}{16\pi^2}\left\{<\left|\vec{i}\,\delta\hat{\varepsilon}(\vec{q}_s,\Omega)\,\vec{f}\right|^2>+\left|<\vec{i}\,\delta\hat{\varepsilon}(\vec{q}_s,\Omega)\vec{f}>\right|^2(S(\vec{q}_s)-1)\right\} \quad (5)$$

Here symbol <...> denotes an averaging over the orientations of particles, in the case of spherical particles an averaging is absent and expression (5) reduces to that obtained in paper [5]; $S(\vec{q}_s)$ is so-called structure factor which describes the influence of the correlations in the particle positions, N is a concentration of the particles. Below we shall use Percus-Yevick approximation [4] to find $S(\vec{q}_s)$ (the expression for $S(\vec{q}_s)$ is given, for example, in [5]).

III. DIRECTOR CONFIGURATION

To proceed further one needs the director profile $\vec{n}(\vec{r})$. For some cases it was already found [5-7], for some cases we present director field here (the details of director profile calculations will be published elsewhere).

Below we summarize the results for director configurations:

1) spherical particles with weak ($\xi=\frac{WR}{K}<<1$) homeotropic director anchoring on the particle surface [5]. Here W is anchoring energy, R is the particle's radius, K is the elastic constant of LC. Electric field E strength is characterized by the parameter $x_R = R\sqrt{\frac{\varepsilon_a E^2}{4\pi K}}$;

2) spherical particles with weak planar director anchoring. Director distribution is given by the same formulas as in [5] multiplied by (-1).

3) spherical particles with strong homeotropic director anchoring [6];

4) cylindrical particles with weak homeotropic or planar anchoring and particle length much greater than its radius R . In the absence of the external electric field the director profile was found in paper [7];

5) cylindrical particles with strong ($\xi = \infty$) homeotropic director anchoring on the particle surface. In the cylindrical coordinate system with z-axis directed along the cylinder axis a director field takes the form $\vec{n} = (\cos\Phi(r,\varphi), \sin\Phi(r,\varphi), 0)$, where

$$\Phi(r,\varphi) = \frac{1}{2}\arctan\frac{b+u}{v} + \frac{1}{2\pi}\int_0^{\pi}\arctan\frac{\cos p - u}{v}dp, \tag{6}$$

$$u = \frac{1}{2}\left(\left(\frac{r}{R}\right)^2 + \left(\frac{R}{r}\right)^2\right)\cos(2\varphi),\ v = \frac{1}{2}\left(\left(\frac{r}{R}\right)^2 - \left(\frac{R}{r}\right)^2\right)\sin(2\varphi), \tag{7}$$

$b = \frac{1}{2}\left(\left(\frac{a}{R}\right)^2 + \left(\frac{R}{a}\right)^2\right)$, $\Phi(r,\varphi)$ - is the angle between director and $\vec{n}_0 = (1,0,0)$, a - is the distance between the disclination loop and the cylinder axis (under the strong anchoring conditions on the particle surface we found the disclination loops in the director field near the particle surface).

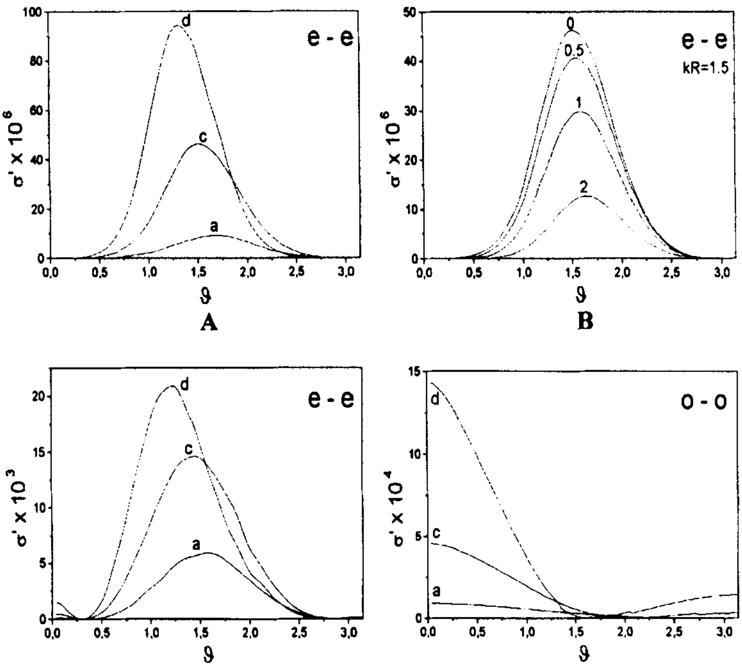
6) cylindrical particles with strong planar anchoring ($\xi = \infty$) on the particle surface. We have now $\vec{n} = (\cos\Phi(r,\varphi), \sin\Phi(r,\varphi), 0)$, where

$$\Phi(r,\varphi) = -\frac{1}{2}\arctan\frac{b-u}{v} + \frac{1}{2\pi}\int_0^{\pi}\arctan\frac{\cos p - u}{v}dp \tag{8}$$

IV. LIGHT SCATTERING CROSS-SECTIONS

In the following plots we show the results of numerical calculations of light scattering differential cross section per particle, $\sigma' \equiv \frac{1}{NR^2}\frac{d\sigma}{d\Omega}$, for the mentioned above director configurations.

Scattering cross-section σ' versus the scattering angle ϑ is shown on Figure 2 for spherical particles. It is necessary to note that only the e-e scattering takes place if the director anchoring on the particle surface is weak. At the strong director anchoring the e-e and o-o types of light scattering take place.



C

FIGURE 2. Spherical particle: A - weak homeotropic (or planar) anchoring ($\xi = 0.1$), $kR = 1 - a, 1.5 - c, 2 - d$; B - nematic in electric field, weak anchoring, $x_R = 0 - 0, 0.5 - 0.5, 1 - 1, 2 - 2$; C - strong homeotropic anchoring ($\xi = \infty$).

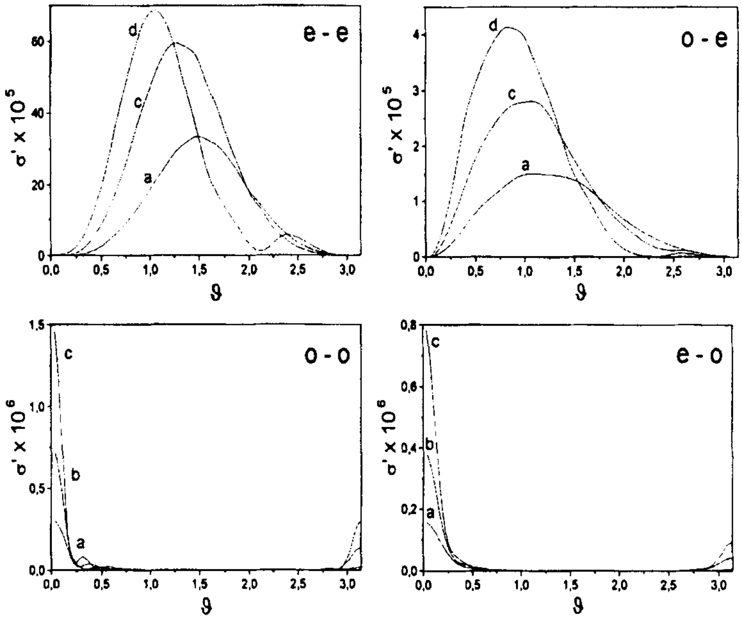
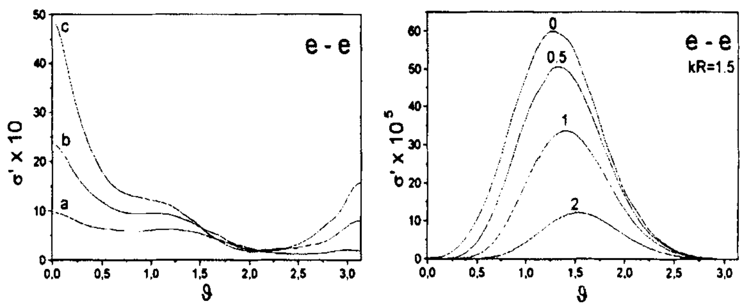


FIGURE 3. Cylindrical particle: weak homeotropic (or planar) anchoring ($\xi = 0.1$), $kR = 1 - a, 1.25 - b, 1.5 - c, 2 - d$.



A **B**

FIGURE 4. Cylindrical particles: A - strong planar anchoring ($\xi = \infty$); B - nematic in electric field, weak ($\xi = 0.1$) anchoring, $x_R = 0 - 0, 0.5 - 0.5, 1 - 1, 2 - 2$.

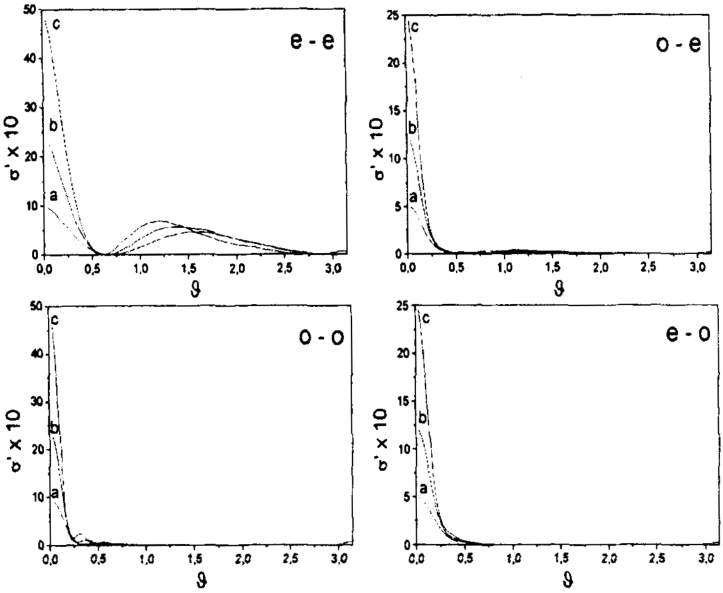


FIGURE 5. Cylindrical particles: strong homeotropic anchoring ($\xi = \infty$).

The values σ' versus ϑ for cylindrical particles are shown on Figure 3 in the case of weak director anchoring, on Figure 4 in the case of strong planar director anchoring (as an example only the e-e type of scattering is shown) and in the case of electric field at the weak director anchoring, on Figure 5 in the case of strong homeotropic director anchoring.

V. CONCLUSIONS

1. The character of the angular dependence of light scattering cross-section depends on the type (e-e,e-o,o-e,o-o) of light scattering and

is caused by the peculiarities of director configuration near the particle.

2. At the weak director anchoring on the particle surface the light scattering differential cross-section practically does not depend on the type of director anchoring (planar or homeotropic).
3. At the strong director anchoring there is small difference in the angular dependence of the scattering cross-sections for planar and homeotropic boundary conditions on surface of long cylindrical particles. It can be explained by the fact that dielectric susceptibility averaged over the volume near the particle has practically the same value for both planar and homeotropic anchoring.
4. At the weak director anchoring the differential cross-section per unit volume of the scattering particle has approximately the same value for both spherical and cylindrical particles, and is essentially greater for cylindrical particles at the strong director anchoring.
5. Increase of the anchoring energy from $\xi = 0.1$ to $\xi = \infty$ results in the differential cross-section increase by 4-5 orders of magnitude.
6. At the weak director anchoring the large angle ($\vartheta \sim 1$) light scattering dominates over the small angle scattering ($\vartheta < 0{,}2$). At the strong anchoring the small angle scattering cross-section has approximately the same value as the large angle scattering cross-section in the case of spherical particles, and significantly (by one order of magnitude) exceeds the last in the case of cylindrical particles. It can be understood if one takes into account that the small angle scattering is determined by the director distortions at great distances from particle center. Therefore the increase of ξ leads to the increase of distorted region characteristic length. For

long cylindrical particles the director distortion decreases with the distance from particle more slowly than for spherical particles of the same volume.

7. In the external electric field E the light scattering differential cross-section decreases lightly faster than $1/E$.

Acknowledgements
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